EFFECTIVE ACTION OF COMPOSITE FIELDS FOR GENERAL GAUGE THEORIES IN BLT–COVARIANT FORMALISM

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Abstract

The gauge dependence of the effective action of composite fields for general gauge theories in the framework of the quantization method by Batalin, Lavrov and Tyutin is studied. The corresponding Ward identities are obtained. The variation of composite fields effective action is found in terms of new set of operators depending on composite field. The theorem of the on-shell gauge fixing independence for the effective action of composite fields in such formalism is proven. Brief discussion of gravitational-vector induced interaction for Maxwell theory with composite fields is given.

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1 INTRODUCTION

The advanced methods of covariant quantization for general gauge theories are based either on the BRST symmetry principle realized in the well-known quantization scheme by Batalin and Vilkovisky [1] or on the extended BRST symmetry principle recently realized within the quantization method by Batalin, Lavrov and Tyutin (BLT) [2]. The various aspects and properties of the gauge field theory within the BV quantization have been under study for quite a long time by now and may be considered as well-known ones (see, for example, reviews [3,4]). On the same time the study of properties as well as various possibilities of interpretation and generalizations of gauge theories in the BLT quantization [2] has been started quite recently [5-16]. Following the line of the research of refs.[5-16] present paper is devoted to the study of one of the central problems arising in quantum gauge field theory within the Lagrangian formalism, i.e. gauge dependence of generating functionals of Green's functions in general gauge theories with composite fields.

Our interest in consideration of composite fields within the BLT- formalism is caused by to a number of reasons. First of all, since the work [17] (for a review, see [18]) where the formalism to study the effective action (EA) for composite fields has been introduced such EA is often used to discuss the dynamical chiral symmetry breaking phenomenon in different models using for example Schwinger-Dyson equations. Second, in four - fermion models [19] the fermions form the composite boundstates which may play the role of Higgs field for discussion of dynamical symmetry breaking in the Standard Model (see [20] and references therein). Third, in the models of inflationary Universe the composite

boundstate may play the role of the inflaton. Finally, the Wilson effective action for composite fields maybe extremely important in recent studies on the exact results in SUSY theories (for a review, see [21]).

The paper is organized as follows. In Sec.2 we give a short review of the BLT formalism. In Sec.3 we derive the Ward identities for effective action of composite fields in general gauge theories in the framework of the quantization method by Batalin, Lavrov and Tyutin. Sec.4 is devoted to the study of gauge dependence structure for composite fields EA. In Sec.5 the example of this effective action is considered. Concluding remarks are given in Sec.6. In our paper we use the notations of refs.[2].

2   BATALIN - LAVROV - TYUTIN QUANTIZATION

In this section we give a short review of the main features of BLT–quantization method for general gauge theories. In order to do this we start from the definition of general gauge theories.

Let us consider the theory of fields $A^i (i = 1, 2, ..., n, \varepsilon(A^i) = \varepsilon_i)$ for which the initial classical action $\mathcal{S}(A)$ is invariant under the gauge transformations $\delta A^i = R^i_\alpha(A)\xi^\alpha$:

$$\mathcal{S}_{,i}(A)R^i_\alpha(A) = 0,$$

$$\alpha = 1, 2, ..., m, 0 < m < n, \ \ \varepsilon(\xi^\alpha) = \varepsilon(\alpha), \qquad (1)$$

where $\varepsilon(\xi^\alpha)$ are arbitrary functions, and the $R^i_\alpha(A)$ are generators of gauge transformations. We suppose the set $R^i_\alpha(A)$ being the linearly independent (case of irreducible theories) and complete. One can say that as a consequence

of the condition of completeness the algebra of generators has the following general form:

$$R^i_{\alpha,j}(A)R^j_\beta(A) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} R^i_{\beta,j}(A)R^j_\alpha(A) =$$

$$-R^i_\gamma(A)F^\gamma_{\alpha\beta}(A) - \mathcal{S}_{,i}(A)M^{ij}_{\alpha\beta}(A), \qquad (2)$$

where $M^{ij}_{\alpha\beta}$ satisfy the conditions

$$M^{ij}_{\alpha\beta} = -(-1)^{\varepsilon_i \varepsilon_j} M^{ji}_{\alpha\beta} = -(-1)^{\varepsilon_\alpha \varepsilon_\beta} M^{ij}_{\beta\alpha}$$

The gauge theories whose generators satisfy Eq.(2) are called general gauge theories. As it has already been mentioned, covariant quantization of such theories (as well as reducible ones) in the framework of a standard BRST symmetry in modern form has been proposed by Batalin and Vilkovisky [1].

To construct the BLT–quantization scheme it is necessary to introduce the total configuration space ϕ^A. For irreducible theories the total configuration space ϕ^A has the following form

$$\phi^A = (A^i, \ B^\alpha, \ C^{\alpha a}), \quad \varepsilon(\phi^A) = \varepsilon_A.$$

Here $C^{\alpha a}$ is $Sp(2)$– doublet of ghost ($a = 1$) and antighost ($a = 2$) fields (Faddeev-Popov fields), B^α are auxiliary fields

$$\varepsilon(B^\alpha) = \varepsilon_\alpha, \quad \varepsilon(C^{\alpha a}) = \varepsilon_\alpha + 1.$$

For reducible theories the complete set of field variables ϕ^A includes also pyramids of the ghosts, the antighosts and the Lagrange multipliers which are combined into irreducible representations of the $Sp(2)$–group (for more detailed discussion, see [2]).

For each field ϕ^A of the total configuration space one introduces three kinds of antifields ϕ^*_{Aa}, $\varepsilon(\phi^*_{Aa}) = \varepsilon_A + 1$ and $\bar\phi_A, \varepsilon(\bar\phi_A) = \varepsilon_A$. The antifields ϕ^*_{Aa}

maybe treated as sources of BRST and antiBRST transformations, while $\bar{\phi}_A$ corresponds to the source of their combined transformation.

On the space of fields ϕ^A and antifields ϕ^*_{Aa} one defines odd symplectic structures $(\, , \,)^a$ called the extended antibrackets

$$(F,G)^a \equiv \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_{Aa}} - (F \leftrightarrow G) \, (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \tag{3}$$

The derivatives with respect to fields are understood as right and those with respect to antifields as left.

The extended antibrackets have the following properties

$$\varepsilon((F,G)^a) = \varepsilon(F) + \varepsilon(G) + 1 \,,$$

$$(F,G)^a = -(G,F)^a(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \,,$$

$$(F,GH)^a = (F,G)^a H + (F,H)^a G(-1)^{\varepsilon(G)\varepsilon(H)} \,,$$

$$((F,G)^{\{a},H)^{b\}}(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \text{cycl.perm.}(F,G,H) \equiv 0 \,, \tag{4}$$

where curly brackets denote symmetrization with respect to the indices a, b of the $Sp(2)$ group. The last relations are the Jacobi identities for the extended antibrackets. In particular, for any boson functional S, $\varepsilon(S) = 0$, one can establish that

$$((S,S)^{\{a},S)^{b\}} \equiv 0$$

The operators V^a, Δ^a are introduced as following

$$V^a = \varepsilon^{ab} \, \phi^*_{Ab} \frac{\delta}{\delta \bar{\phi}_A}, \quad \varepsilon^{ab} = -\varepsilon^{ba}, \varepsilon^{12} = 1, \tag{5}$$

$$\Delta^a = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi^*_{Aa}}, \tag{6}$$

Here the subscript "l" denotes the left derivative with respect to field. It maybe shown that the algebra of operators (5),(6) has the form

$$V^{\{a}V^{b\}} = 0, \quad \Delta^{\{a}\Delta^{b\}} = 0, \quad \Delta^{\{a}V^{b\}} + V^{\{a}\Delta^{b\}} = 0. \tag{7}$$

The action of the operators V^a (4) upon the extended antibrackets is given by the relations

$$V^{\{a}(F, G)^{b\}} = (V^{\{a}F, G)^{b\}} - (-1)^{\varepsilon(F)}(F, V^{\{a}G)^{b\}}. \tag{8}$$

The basic object of BLT scheme is a boson functional $S = S(\phi, \phi_a^*, \bar{\phi})$, satisfying the following generating equations

$$\frac{1}{2}(S, S)^a + V^a S = i\hbar \Delta^a S \tag{9}$$

with the boundary condition

$$S|_{\phi_a^* = \bar{\phi} = \hbar = 0} = \mathcal{S}(A). \tag{10}$$

It should be noted that Eqs.(9) are compatible. The simplest way to establish this fact is to rewrite Eqs.(9) in an equivalent form of linear differential equations

$$\bar{\Delta}^a \exp\left\{\frac{i}{\hbar}S\right\} = 0. \tag{11}$$

¿From (7) it follows that the operators $\bar{\Delta}^a$ in (11) possess the properties

$$\bar{\Delta}^{\{a}\bar{\Delta}^{b\}} = 0, \tag{12}$$

and therefore Eqs.(9) are compatible.

The quantum action $S_{ext} = S_{ext}(\phi, \phi_a^*, \bar{\phi})$ for constructing of Feynman rules in the BLT scheme is introduced as

$$\exp\left\{\frac{i}{\hbar}S_{ext}\right\} = \exp\left\{-i\hbar\hat{T}(F)\right\}\exp\left\{\frac{i}{\hbar}S\right\}, \tag{13}$$

where the operator $\hat{T}$ has the form

$$\hat{T}(F) = \frac{1}{2}\varepsilon_{ab}[\bar{\Delta}^b, [\bar{\Delta}^a, F]_-]_+, \tag{14}$$

$F = F(\phi, \phi_a^*, \bar{\phi})$ is the boson functional fixing a concrete choice of admissible gauge. Note that the operator $\hat{T}$ commutes with $\bar{\Delta}^a$ for arbitrary F

$$[\hat{T}, \bar{\Delta}^a]_- = 0 \qquad (15)$$

and hence S_{ext} satisfies the equations (9)

$$\bar{\Delta}^a \exp\left\{\frac{i}{\hbar} S_{ext}\right\} = 0. \qquad (16)$$

The generating functional of the Green's functions $Z(J)$ is defined as

$$Z(J) = \mathcal{Z}(J, \phi_a^*, \bar{\phi})|_{\phi_a^* = \bar{\phi} = 0}, \qquad (17)$$

where the extended generating functional $\mathcal{Z}(J, \phi_a^*, \bar{\phi})$ has the form

$$\mathcal{Z}(J, \phi_a^*, \bar{\phi}) = \int d\phi \exp\left\{\frac{i}{\hbar}[S_{ext}(\phi, \phi_a^*, \bar{\phi}) + J_A \phi^A]\right\}. \qquad (18)$$

It is not difficult to show that the integrand in Eq.(18) for $J = 0$ is invariant under the following global transformations

$$\delta\phi^A = \frac{\delta S_{ext}}{\delta\phi_{Aa}^*}\mu_a, \ \delta\phi_{Aa}^* = 0, \ \delta\bar{\phi}_A = -\mu_a \varepsilon^{ab} \phi_{Ab}^*, \qquad (19)$$

where μ_a is the $Sp(2)$–doublet of constant Grassmann parameters $(\varepsilon(\mu_a) = 1)$. These transformations are nothing but the extended BRST ones in the BLT quantization.

Note that from the definitions (17),(18) and Eqs.(19) it follows that the extended BRST transformations for the generating functional $Z(J)$ have the form

$$\delta\phi^A = \frac{\delta S_{ext}}{\delta\phi_{Aa}^*}\Bigg|_{\phi^* = \bar{\phi} = 0}\mu_a. \qquad (20)$$

The symmetry of the vacuum functional $Z(0)$ under the transformations (20) permits to show the independence of S–matrix from the choice of a gauge(within the BLT formalism [2].)

As a consequence of the fact that S_{ext} satisfies the generating equations (16) one can write the Ward identities within the BLT formalism. For the extended generating functional $\mathcal{Z} = \mathcal{Z}(J, \phi_a^*, \bar\phi)$ (18) these identities have the form

$$\left(J_A \frac{\delta}{\delta \phi_{Aa}^*} - \varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \bar\phi_A} \right) \mathcal{Z} = 0. \tag{21}$$

Introducing in a standard manner the generating functional of the vertex functions Γ,

$$\Gamma(\phi, \phi_a^*, \bar\phi) = \frac{\hbar}{i} \ln \mathcal{Z}(J, \phi_a^*, \bar\phi) - J_A \phi^A, \tag{22}$$

$$\phi^A = \frac{\hbar}{i} \frac{\delta \ln \mathcal{Z}(J, \phi_a^*, \bar\phi)}{\delta J_A},$$

we obtain the Ward identities

$$\frac{1}{2}(\Gamma, \Gamma)^a + V^a \Gamma = 0 \tag{23}$$

These Ward identities will be used in the study of the gauge dependence of EA.

3   WARD IDENTITIES

In this section we derive the Ward identities for general gauge theories with composite fields in the framework of BLT covariant quantization.

Let us introduce the composite fields

$$\sigma^m(\phi) = \sum_{n=2}^{} \frac{1}{n!} \Lambda_{A_1 \ldots A_n}^m \phi^{A_1} \ldots \phi^{A_n}, \quad \varepsilon(\sigma^m) \equiv \varepsilon_m. \tag{24}$$

Using the quantum action $S_{ext} = S_{ext}(\phi, \phi_a^*, \bar\phi)$ (13) we define the generating functional $Z(J, \phi_a^*, \bar\phi, L)$ for the composite fields $\sigma^m(\phi)$ as following

$$Z(J, \phi_a^*, \bar\phi, L) = \int d\phi \exp \left\{ \frac{i}{\hbar} [S_{ext}(\phi, \phi_a^*, \bar\phi) + J_A \phi^A + L_m \sigma^m(\phi)] \right\} =$$

$$= \exp \left\{ \frac{i}{\hbar} W(J, \phi_a^*, \bar{\phi}, L) \right\}, \tag{24}$$

where $W(J, \phi_a^*, \bar{\phi}, L)$ is the generating functional of the connected correlation functions for composite fields, and L_m are sources for σ^m.

The Ward identities for general gauge theories with composite fields are obtained as a consequence of the fact that S_{ext} satisfies the generating equations (16). To do this one can multiply Eqs.(16) on functional

$$\exp \left\{ \frac{i}{\hbar} [J_A \phi^A + L_m \sigma^m(\phi)] \right\}$$

and integrate over fields ϕ^A. Then we have the identities

$$\int d\phi \exp \left\{ \frac{i}{\hbar} [J_A \phi^A + L_m \sigma^m(\phi)] \right\} \bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S_{ext}(\phi, \phi_a^*, \bar{\phi}) \right\} = 0. \tag{26}$$

Carrying out integration by parts in (26) one derives the Ward identities for functional $Z(J, \phi_a^*, \bar{\phi}, L)$

$$\left\{ \left(J_A + L_m \sigma_{,A}^m \left(\frac{\hbar}{i} \frac{\delta}{\delta J} \right) \right) \frac{\delta}{\delta \phi_{Aa}^*} - V^a \right\} Z(J, \phi_a^*, \bar{\phi}, L) = 0. \tag{27}$$

For the functional $W(\phi, \phi_a^*, \bar{\phi}, L)$ the identities (27) are

$$\left\{ \left(J_A + L_m \sigma_{,A}^a \left(\frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \right) \frac{\delta}{\delta \phi_{Aa}^*} - V^a \right\} W(J, \phi_a^*, \bar{\phi}, L) = 0. \tag{28}$$

Here we have used the notations

$$\sigma_{,A}^m(\phi) \equiv \frac{\delta \sigma^m(\phi)}{\delta \phi^A}.$$

Let us introduce the generating functional of vertex functions (effective action) for composite fields $\Gamma = \Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma)$ by the rule

$$\Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma) = W(J, \phi_a^*, \bar{\phi}, L) - J_A \phi^A - L_m(\Sigma^m + \sigma^m(\phi)), \tag{29}$$

where

$$\phi^A = \frac{\delta W(J, \phi_a^*, \bar{\phi}, L)}{\delta J_A},$$

$$\Sigma^m = \frac{\delta W(J, \phi_a^*, \bar{\phi}, L)}{\delta L_m} - \sigma^m \left(\frac{\delta W}{\delta J} \right). \tag{30}$$

¿From definitions (29),(30) it follows that

$$\frac{\delta \Gamma}{\delta \phi^A} = -J_A - L_m \sigma_{,A}^m \left(\frac{\delta W}{\delta J} \right), \frac{\delta \Gamma}{\delta \Sigma^m} = -L_m \tag{31}$$

Using expressions (28) and definition (29) one can obtain the Ward identities for $\Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma)$ in the form

$$\frac{1}{2}(\Gamma, \Gamma)^a + V^a \Gamma + \frac{\delta \Gamma}{\delta \Sigma^m} \left(\sigma_{,A}^m(\hat{\phi}) - \sigma_{,A}^m(\phi) \right) \frac{\delta \Gamma}{\delta \phi_{Aa}^*} = 0, \tag{32}$$

where

$$\hat{\phi}^A = \phi^A + i\hbar (G''^{-1})^{A\alpha} \frac{\delta_l}{\delta \Phi^\alpha}, \tag{33}$$

$$\Phi^\alpha = (\phi^A, \Sigma^m), \quad G''_{\alpha\beta} = \frac{\delta_l E_\beta}{\delta \Phi^\alpha}, \tag{34}$$

$$E_\alpha = \left(\frac{\delta \Gamma}{\delta \phi^A} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma_{,A}^m(\phi), \frac{\delta \Gamma}{\delta \Sigma^m} \right). \tag{35}$$

These identities are useful in various aspects, in particularly, in the study of the gauge dependence. They generalize the corresponding Ward identities of Section 2 for the case of composite fields.

4   THE GAUGE DEPENDENCE

In this section we discuss the gauge dependence of generating functionals Z, W, Γ for general gauge theories with composite fields. The derivation of this dependence is based on the fact that any variation of gauge functional $F \to F + \delta F$ leads to variation of action S_{ext} (13) and functional Z (25). One can easily check that the variation of action can be expressed in the form

$$\delta \left(\exp \left\{ \frac{i}{\hbar} S_{ext} \right\} \right) = -i\hbar \hat{T}(\delta \hat{X}) \exp \left\{ \frac{i}{\hbar} S_{ext} \right\} \tag{36}$$

with some operator $\delta\hat{X}$ of first order with respect to δF. For our purposes it is not important to know explicit expression of operator $\delta\hat{X}$ through δF. Note only that one can always present the operator $\delta\hat{X}$ in the following way

$$\delta\hat{X}\left(\phi,\phi_a^*,\bar{\phi};\frac{\delta}{\delta\phi},\frac{\delta}{\delta\phi_a^*},\frac{\delta}{\delta\bar{\phi}},\right) = \sum_{n,m,l=0}\left(\delta X_{B_1b_1...B_mb_mC_1...C_l}^{A_1...A_n}(\phi,\phi_a^*,\bar{\phi})\right.$$

$$\left.\frac{\delta}{\delta\phi^{A_1}}\cdots\frac{\delta}{\delta\phi^{A_n}}\frac{\delta}{\delta\phi_{B_1b_1}^*}\cdots\frac{\delta}{\delta\phi_{B_mb_m}^*}\frac{\delta}{\delta\bar{\phi}_{C_1}}\cdots\frac{\delta}{\delta\bar{\phi}_{C_l}}\right). \tag{37}$$

¿From Eqs.(13), (15), (26) it follows that variation of functional Z can be written as

$$\delta Z(J,\phi_a^*,\bar{\phi},L) =$$

$$= \int d\phi\exp\left\{\frac{i}{\hbar}[J_A\phi^A + L_m\sigma^m(\phi)]\right\}(-i\hbar\hat{T}(\delta\hat{X}))\exp\left\{\frac{i}{\hbar}S_{ext}(\phi,\phi_a^*,\bar{\phi})\right\} =$$

$$= -\frac{i\hbar}{2}\varepsilon_{ab}\int d\phi\exp\left\{\frac{i}{\hbar}[J_A\phi^A + L_m\sigma^m(\phi)]\right\}\bar{\Delta}^b\bar{\Delta}^a\delta\hat{X}\exp\left\{\frac{i}{\hbar}S_{ext}(\phi,\phi_a^*,\bar{\phi})\right\}. \tag{38}$$

Carrying out integration by parts in the functional integral (38) one can rewrite the variation of Z in the form

$$\delta Z(J,\phi_a^*,\bar{\phi},L) =$$

$$= \frac{i}{2\hbar}\varepsilon_{ab}\hat{q}^b\hat{q}^a\delta\hat{X}\left(\frac{\hbar}{i}\frac{\delta}{\delta J},\phi_a^*,\bar{\phi};\frac{1}{i\hbar}\left(J + L\sigma,\left(\frac{\hbar\delta}{i\delta J}\right)\right),\frac{\delta}{\delta\phi_a^*},\frac{\delta}{\delta\bar{\phi}}\right)Z(J,\phi_a^*,\bar{\phi},L), \tag{39}$$

where $\hat{q}^a$ stands for an operatorial $Sp(2)$-doublet

$$\hat{q}^a = -\left[J_A + L_m\sigma_{,A}^m\left(\frac{\hbar}{i}\frac{\delta}{\delta J}\right)\right]\frac{\delta}{\delta\phi_{Aa}^*} + V^a, \tag{40}$$

which is directly verified to satisfy the relations

$$\hat{q}^{\{a}\hat{q}^{b\}} = 0 \tag{41}$$

From the relations (39),(40) taking into account of the Ward identities (27) for Z and the fact that

$$\delta Z = \frac{i}{\hbar}\delta W\ Z,$$

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it follows the expression for the variation of functional $W(J, \phi_a^*, \bar{\phi}, L)$:

$$\delta W(J, \phi_a^*, \bar{\phi}, L) = \frac{1}{2}\varepsilon_{ab}\hat{Q}^b\hat{Q}^a < \delta\hat{X} > , \qquad (42)$$

where the operators $\hat{Q}^a$ are related to $\hat{q}^a$ through a unitary transformation

$$\hat{Q}^a = \exp\{-\frac{i}{\hbar}W\}\hat{q}^a\exp\{\frac{i}{\hbar}W\}$$

and have the form

$$\hat{Q}^a = -\left[J_A + L_m\sigma_{,A}^m\left(\frac{\delta W}{\delta J} + \frac{\hbar}{i}\frac{\delta}{\delta J}\right)\right]\frac{\delta}{\delta\phi_{Aa}^*} + V^a . \qquad (43)$$

As a consequence of the Eq.(41) $\hat{R}^a$ possess the following properties

$$\hat{Q}^{\{a}\hat{Q}^{b\}} = 0 . \qquad (44)$$

In (42), the notation $< \delta\hat{X} >$ is used for the vacuum expectation value of the operator $\delta\hat{X}$

$$< \delta\hat{X} >=$$
$$\delta\hat{X}\left(\frac{\delta W}{\delta J} + \frac{\hbar\delta}{i\delta J}, \phi_a^*, \bar{\phi}; \frac{1}{i\hbar}\left(J + L\sigma, \left(\frac{\delta W}{\delta J} + \frac{\hbar\delta}{i\delta J}\right)\right), \frac{\delta}{\delta\phi_a^*} + \frac{i}{\hbar}\frac{\delta W}{\delta\phi_a^*}, \frac{\delta}{\delta\bar{\phi}} + \frac{i}{\hbar}\frac{\delta W}{\delta\bar{\phi}}\right) .$$

Let us find the expression for $\delta\Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma)$. To this end, we must study some differential consequences from the Ward identities for Z, W and use the following observations that are a consequence of definitions (29)-(31). Namely,

$$\delta W = \delta\Gamma ,$$

$$\frac{\delta}{\delta\phi_a^*}\bigg|_{J,L} = \frac{\delta}{\delta\phi_a^*}\bigg|_{\phi,\Sigma} + \frac{\delta\phi}{\delta\phi_a^*}\frac{\delta_l}{\delta\phi}\bigg|_{\phi_a^*,\Sigma} + \frac{\delta\Sigma}{\delta\phi_a^*}\frac{\delta_l}{\delta\Sigma}\bigg|_{\phi,\phi_a^*} ,$$

$$V_{|J,L}^a = V_{|\phi,\Sigma}^a + V^a\phi\frac{\delta_l}{\delta\phi}\bigg|_{\bar{\phi},\Sigma} + V^a\Sigma\frac{\delta_l}{\delta\Sigma}\bigg|_{\phi,\bar{\phi}} . \qquad (45)$$

Next, differentiating the Ward identities for Z (27) with respect to the sources J and L, then rewriting these relations for the functional W and transforming

the latter with allowance for Eqs.(29)-(31) we obtain

$$\hat{Q}^a \phi^A_{|L,J} = \frac{\delta\Gamma}{\delta\phi^*_{Aa}}(-1)^{\varepsilon_A}+$$

$$\frac{i}{\hbar}\left(\phi^A \frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}(-1)^{\varepsilon_A} - \frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\phi^A\right),$$

$$\hat{Q}^a\Sigma^n_{|L,J} = \left(\sigma^n_{,B}(\hat{\phi}) - \sigma^n_{,B}(\phi)\right)\frac{\delta\Gamma}{\delta\phi^*_{Ba}}(-1)^{\varepsilon_n} + .$$

$$\frac{i}{\hbar}\left(\left(\Sigma^n + \sigma^n(\phi)\right)\frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}(-1)^{\varepsilon_n}-\right.$$

$$\frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\left(\Sigma^n + \sigma^n(\phi)\right)\Big)-$$

$$\frac{i}{\hbar}(-1)^{\varepsilon_n}\sigma^n_{,C}(\phi)\left(\phi^C \frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}} - (-1)^{\varepsilon_C}\frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\phi^C\right)+$$

$$(-1)^{\varepsilon_n}\left(\sigma^n_{,C}(\phi)\frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})(G''^{-1})^{C\alpha}\frac{\delta_l}{\delta\Phi^\alpha}\frac{\delta\Gamma}{\delta\phi^*_{Ba}}(-1)^{\varepsilon_C\varepsilon_B}-\right.$$

$$(-1)^{\varepsilon_n\varepsilon_B}\frac{\delta\Gamma}{\delta\Sigma^m}\sigma^m_{,B}(\hat{\phi})\sigma^n_{,C}(\phi)(G''^{-1})^{C\alpha}\frac{\delta_l}{\delta\Phi^\alpha}\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\Big). \qquad (46)$$

Taking into account the relations (43),(45)-(46), we have the final representation for the variation of the effective action with composite fields

$$\delta\Gamma(\phi, \phi^*_a, \bar{\phi}, \Sigma) = \frac{1}{2}\varepsilon_{ab}\hat{s}^b\hat{s}^a << \delta\hat{X} >> , \qquad (47)$$

where we have introduced the notations

$$\hat{s}^a = (\Gamma, \ )^a + V^a + (-1)^{\varepsilon_m}\left(\left(\sigma^m_{,A}(\hat{\phi}) - \sigma^m_{,A}(\phi)\right)\frac{\delta\Gamma}{\delta\phi^*_{Aa}}\right)\frac{\delta_l}{\delta\Sigma^m} + \frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,A}(\hat{\phi}) - \sigma^m_{,A}(\phi)\right)\frac{\delta}{\delta\phi^*_{Aa}} -$$

$$\frac{i}{\hbar}\left[\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\Phi^\alpha\right) - (-1)^{\varepsilon_\alpha}\Phi^\alpha\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\right)\right]\frac{\delta_l}{\delta\Phi^\alpha} -$$

$$\frac{i}{\hbar}\left[\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\sigma^n(\phi)\right) - (-1)^{\varepsilon_n}\sigma^n(\phi)\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\right)\right]\frac{\delta_l}{\delta\Sigma^n} +$$

$$\frac{i}{\hbar}(-1)^{\varepsilon_n}\sigma^n_{,C}(\phi)\left[\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\phi^C\right)(-1)^{\varepsilon_C} - \phi^C\frac{\delta\Gamma}{\delta\Sigma^m}\left(\sigma^m_{,B}(\hat{\phi})\frac{\delta\Gamma}{\delta\phi^*_{Ba}}\right)\right]\frac{\delta_l}{\delta\Sigma^n} +$$

$$(-1)^{\varepsilon_m}\left[\sigma^m_{,C}(\phi)\frac{\delta\Gamma}{\delta\Sigma^n}\left(\sigma^n_{,A}(\hat{\phi})(G''^{-1})^{C\alpha}\frac{\delta_l}{\delta\Phi^\alpha}\frac{\delta\Gamma}{\delta\phi^*_{Aa}}\right)(-1)^{\varepsilon_C\varepsilon_A} -\right.$$

$$(-1)^{\varepsilon_m \varepsilon_A} \frac{\delta \Gamma}{\delta \Sigma^n} \left(\sigma_{,A}^n(\hat{\phi}) \sigma_{,C}^m(\phi)(G''^{-1})^{C\alpha} \frac{\delta_l}{\delta \Phi^\alpha} \frac{\delta \Gamma}{\delta \phi_{Aa}^*} \right) \Big] \frac{\delta_l}{\delta \Sigma^m} \, , \qquad (48)$$

$$<< \delta \hat{X} >> \equiv$$

$$\equiv \delta \hat{X} \left(\hat{\phi}, \phi_a^*, \bar{\phi}; \frac{1}{i\hbar} \left(-\Gamma_{,} + \Gamma_{,m} \left[\sigma_{,}^m(\phi) - \sigma_{,}^m(\hat{\phi}) \right] \right), \quad \frac{\delta}{\delta \phi_a^*} + \frac{i\delta\Gamma}{\hbar \delta \phi_a^*}, \frac{\delta}{\delta \bar{\phi}} + \frac{i\delta\Gamma}{\hbar \delta \bar{\phi}} \right) \, , \quad (49)$$

while the action of the operators $\left[(\Gamma, \quad)^a + V^a \right]$ on an arbitrary functional $G = G(\phi, \phi_a^*, \bar{\phi}, \Sigma)$, is understood as follows:

$$\left[(\Gamma, \quad)^a + V^a \right] G \equiv (\Gamma, G)^a + V^a G \equiv \hat{s}_0^a G \, . \qquad (50)$$

The operators $\hat{s}_0^a$ are said to be the generators of the quantum extended BRST transformations without composite fields in the BLT method for general gauge theories, while the operators $\hat{s}^a$ can be considered as a deformation of generators $\hat{s}_0^a$ related to the presence of the composite fields in the theory.

Since the values $\hat{s}^a$ are obtained through a change of variables, i.e. the Legendre transformation, from the operatorial Sp(2)-doublet $\hat{Q}^a$ satisfying Eq.(44), the operators $\hat{s}^a$ must possess the following algebra

$$\hat{s}^a \hat{s}^b = 0 \qquad (51)$$

This is the standard algebra for the extended BRST symmetry. It is surprising that this algebra remains the same in case of an arbitrary composite field. Note,however, that the above operators are defined with the help of composite fields EA which satisfy the Ward identities.

The corresponding algebra of the operators $\hat{s}_0^a$ has the standard form as for $\hat{s}^a$

$$\hat{s}_0^{\{a} \hat{s}_0^{b\}} = 0 \, , \qquad (52)$$

which is a consequences of the algebra of operators V^a in (7), of Jacobi identities for the extended antibrackets (4), of the Leibnitz rule of the action of the V^a on the extended antibrackets (8) and of the Ward identities for $\Gamma(\phi, \phi_a^*, \bar{\phi})$ (23).

It should be noted that the expression for $\delta\Gamma$ (47) has very simple and remarkable form. It is defined through the commutator of the operators $\hat{s}^a$. We expect that the gauge dependence of general gauge theories can be understood also in geometrical terms using, for example, the local BRST cogomology in the same style as in refs.[28].

Note that the expression for $\delta\Gamma$ can be rewritten in the following equivalent form (cf. Ref.5) with the help of systematic use of Ward identities for Γ (32) and their differential consequences:

$$\delta\Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma) = \frac{\delta\Gamma}{\delta\Phi^\alpha}W^\alpha + \phi_{Aa}^* D^{Aa} \tag{53}$$

with the fully definite functionals W^α and D^{Aa} depending on all the variables $\phi, \phi_a^*, \bar{\phi}, \Sigma$.

¿From Eq.(53) we get the following Theorem: the generating functional of vertex functions $\Gamma(\phi, \phi_a^*, \bar{\phi}, \Sigma)$ in the BLT quantization method of general gauge theories with composite fields does not depend on the gauge on its extremals which are defined as

$$\frac{\delta\Gamma}{\delta\Phi^\alpha} = 0 \tag{54}$$

and the hypersurface defined by conditions

$$\phi_{Aa}^* = 0. \tag{55}$$

It is useful to compare this result with the gauge dependence of generating functional of vertex functions with composite fields in the BV–quantization method. This problem has been investigated in ref.[22] with the following result- generating functional of vertex functions with composite fields is gauge independent only on its extremals.Here we have a more complicated case.

With the help of Eq.(52) one can develop another point of view on the problem of gauge dependence. Namely, the variation of effective action in the BLT quantization method for general gauge theories with composite fields is proportional to the commutator of the operators $\hat{s}^a$ which act on the corresponding variation $<< \delta \hat{X} >>$.

Notice that an essential feature of the our proof is the assumption of existence of "deep" gauge invariant regularization preserving the Ward identities and permitting to use their differential consequences. Then we expect that the corresponding completely renormalized generating functionals satisfy the same properties as the nonrenormalizable ones.

5   APPLICATIONS: INDUCED GRAVITATIONAL - VECTOR INTERACTION

Let us discuss some cosmological application of the composite fields effective action in the external gravitational field. Note that the BLT formalism still has not been generalized to the problems with external fields, so this section lies outside of the general study developed in this paper.

There were some discussions recently on the presence of magnetic field in the early Universe. It could be of a primordial origin, and be produced in the inflationary Universe. However with usual Maxwell - type lagrangian it seems to be impossible to produce such the magnetic field [23]. One can consider magnetic field in string cosmology [24], or by addition the terms of form $RA_{\mu}A^{\mu}$ to Maxwell lagrangian. However, such terms break gauge invariance.

The way out maybe found by using the composite fields effective action. Let

us consider the generating functional W for Maxwell theory ($\hbar = 1$):

$$\exp\{i\,W[J,K]\} = \int \mathcal{D}A^a_\mu \exp\left\{i[S + \int d^4x \sqrt{-g}(J_\mu A^\mu + K A_\mu A^\mu)]\right\} \qquad (56)$$

where $S = -\frac{1}{4}\int d^4x \sqrt{-g}F_{\mu\nu}F^{\mu\nu}$, the Landau gauge which is an effective gauge is chosen, the presence of the corresponding gauge breaking term and ghost action is supposed. We don't discuss the renormalization of the vacuum sector (action for external gravitational field) which has been studied in all detail in [25].

Study of the generating functional (56) shows that its renormalization induces few terms. Among of them there is term of the form KR [26]. Using the correspondent non-homogeneous renormalization group equation and explicit one-loop calculations one can show that Maxwell sector is modifying as

$$W[0,K] = -\frac{1}{4}\int d^4x \sqrt{-g}\left\{F_{\mu\nu}F^{\mu\nu} - \frac{1}{(4\pi)^2}KR\ln\frac{|R+K|}{\mu^2} - \frac{12K^2}{(4\pi)^2} + ...\right\}.$$

Hence, the gravitational-vector interaction is induced on quantum level. Such a term maybe relevant to produce the strong enough magnetic field in the early Universe in a consistent way (without breaking of gauge invariance on classical level). Hence, the development of quantization schemes for theories in the external fields is getting quite important.

6   DISCUSSION

In summary, composite fields EA for general gauge theories is investigated in frames of BLT-quantization method (Sp(2) formalism). The new set of operators which depend on composite fields is introduced. Their algebra coincides

with the algebra of extended BRST transformations. The variation of composite fields EA is found in a very simple form, using this new set of operators and Ward identities. The proof of on-shell gauge fixing independence of the composite fields EA is given. Some properties of EA for composite vector fields are briefly discussed.

The importance of the composite fields EA in external background has been mentioned recently (see for example [27]) in connection with the study of the exact potential in supersymmetric YM theories. Hence, it is necessary to generalize the quantization methods existing already also for composite fields EA in an external background. ¿From another side, the connected study of the structure and properties of gauge theories in the presence of external fields is getting of interest due to possible applications. Such an investigation maybe quite non-trivial what follows from results of ref.[30] where Ward identities for gauge theories have been obtained within BV quantization [1] in the presence of external fields. Hence, it is very interesting to generalize the results of this work (Ward identities, Theorem) for the case of external gauge and (or) gravitational background.

Another interesting line of research is related with the study of BRST cohomologies a la ref.[28] in BLT-formalism. Recently, using results of [28] it has been shown that non-renormalizable theories maybe renormalizable in a modern sense [29] (taking into account the infinite number of counterterms). Then it would be of interest to formulate the proof of ref.[29] within BLT formalism (even taking into account composite fields).

7 APPENDIX A

In this Appendix we will give few remarks on Wilson action for composite fermion fields. We consider the theory containing spinors ψ and gauge fields A_μ^a.

The generating functional W (euclidean notations are used) is defined as

$$\exp(-W[J]) = \int \mathcal{D}\phi \exp\{-S_L(\psi, \bar{\psi}, A_\mu) - J\bar{\psi}\psi\}, \qquad (A1)$$

where ϕ is the set of all fields (including ghost) and ghost term and gauge - fixing term are supposed to be present in (A1). The Wilson effective action is defined through the introduction of the infra-red cut-off L, and background field propagators in (A1) are modified (compare with standard) as

$$K_L^{-1} = \int_0^L dt \exp(-tK) \qquad (A2)$$

in the action S_L (for $L \to \infty$ it becomes the standard propagator). The corresponding flow equation for W maybe written.

The Wilson effective action is defined via the Legendre transform

$$\Gamma_L(<\bar{\psi}\psi>) = W[J] - J <\bar{\psi}\psi> \qquad (A3)$$

Such Wilson effective action for composite fields maybe easily applied to study the exact results in SUSY theories (for more details, see [27]). It would be extremely interesting to combine BLT-formalism with Wilson effective action formalism. That would definitely enrich both approaches, but requires quite hard work.

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